B.Sc (Maths) part I paper II

The plane (problem)

problem ① Find the equation of the plane through the point (1,1,0) (1,2,1) & (-2,2,-1)

Sol: — Let the equation of plane be

$$ax+by+cz+d=0 \quad \text{—} ①$$

Since this equation passes through the point (1,1,0) we get

$$a+b+0+d=0 \quad \text{——} ②$$

Subtracting ② from ① we get

$$a(x-1)+b(y-1)+cz=0$$

It will pass through point (1,2,1) & (-2,2,-1)

$$0.a+b+c=0 \quad \text{—} ③$$

$$-3a+b-c=0 \quad \text{—} ④$$

By cross multiplication

$$\frac{a}{-1-1}=\frac{b}{-3-0}=\frac{c}{3}=r \text{ (say)}$$

$\therefore a=-2r,\ b=-3r,\ c=3r$

Substituting the values of

a, b, c in (3) we get

$-2(x-1) - 3(y-1) + 3(z-0) = 0$

$\Rightarrow -2x+2-3y+3+3z=0$

or $2x+3y-3z+5=0$

problem (2) Find the equation of the plane through the points $(2,2,1)$, $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.

Sol:- Let the equation of the required plane be

$$ax+by+cz+d=0 \quad \text{---(1)}$$

It is perpendicular to the given plane $2x+6y+6z=9$

$\therefore aa_1+bb_1+cc_1=0$ (formula)

$\Rightarrow a\cdot 2+b\cdot 6+c\cdot 6=0$

$\Rightarrow 2a+6b+6c=0 \quad \text{---(2)}$

Since the plane (1) passes through the given points $(2,2,1)$ and $(9,3,6)$

Therefore $a\cdot 2+b\cdot 2+c\cdot 1+d=0 \quad \text{---(3)}$

and $a\cdot 9+b\cdot 3+c\cdot 6+d=0 \quad \text{---(4)}$

On subtracting $-7a-b-5c=0$

$\Rightarrow 7a+b+5c=0 \quad \text{---(5)}$

Solving ② and ⑤ by cross multiplication we get

$$2a+6b+6c=0$$

$$7a+b+5c=0$$

$$\frac{a}{30-6} = \frac{b}{42-10} = \frac{c}{2-42}$$

$$\frac{a}{24} = \frac{b}{32} = \frac{c}{-40}$$

$$\Rightarrow \frac{a}{3} = \frac{b}{4} = \frac{c}{-5} = K$$

$$a = 3K,\ b = 4K,\ c = -5K$$

Substituting these values in ③ we get

$$6K+8K-5K+9=0$$

$$9 = -9K$$

Putting values of a, b, c, d in ① we get

$$8Kx+4Ky-5Kz-9K=0$$

$$\Rightarrow 8x+4y-5z-9=0$$

This is required equation.

problem ③ Find the equation of the plane through the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and perpendicular to the plane $5x+3y+6z+8=0$

Soln. — Let the equation of the plane through the line of intersection of planes

$x+2y+3z-4=0$

and $2x+y-z+5=0$ be

$x+2y+3z-4+K(2x+y-z+5)=0$

or $(1+2K)x+(2+K)y+(3-K)z$

$+(5K-4)=0$ ——— (1)

Since it is perpendicular to the given plane $5x+3y+6z+8=0$

therefore $a_1a_2+b_1b_2+c_1c_2=0$ (formula)

or $(1+2K)\cdot 5+(2+K)\cdot 3+(3-K)6=0$

or $7K+29=0$ or $K=-\frac{29}{7}$

Substituting the value of K in (1) we get

$9x+15y-50z+173=0$

This is required eqn. of plane.